



Theoretical study of 2×2 planar array of equilateral triangular patch microstrip antenna on ferrite substrate

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This paper presents theoretical investigations of the radiation properties of a 2×2 element planar array of equilateral triangular patch antenna built up on a magnetized ferrite substrate $\text{Ni}_{1.062}\text{Co}_{0.02}\text{Fe}_{1.948}\text{O}_4$ in the presence of plasma medium. The far zone EM-mode and P-wave radiation fields are derived using vector wave function techniques and pattern multiplication approaches. The results are obtained both in plasma medium and in free space. Computation of maximum field intensity, half power beam width (HPBW), full null beam width (FNBW) and other antenna parameters such as radiation conductance, directivity and quality factor has been made for the plasma and the free space medium. The results of this theoretical analysis suggest that due to ferrite substrate, this planar array antenna can be operated at lower frequency range and there is a need of size of such antenna when designed on NiCoFeO_4 ferrite substrate.

84-40 Hz, 52-40 Fd

Introduction

In recent past, microstrip antennas built up on ferrite substrate have been attracting increasing attention to the numerous scientist and engineers. Ferrite materials have a significant amount of anisotropy at microwave frequencies. This anisotropy gets enhanced by applying external d.c. magnetic field in ferrite materials and brings about non-reciprocal behavior in them.

More recently, with the availability of low loss commercial ferrite materials, ferrite substrates have been used in microstrip antenna applications to provide loading for antenna size reduction. Microstrip antennas on pure dielectric substrate have been extensively analysed in these years due to their advantages over bulky antennas. The band width of such antennas in the frequency range is typically of the order of 1 percent and have not tried at lower ultra high frequency (UHF) due to the size considerations.

Such antennas when mounted on aerospace vehicles enter plasma medium during their travel in space. It has been

pointed out that the radiation properties of microstrip antennas in plasma medium are modified to a great extent due to the generation of electroacoustic waves in addition to electromagnetic waves [1-5].

This paper describes the radiation characteristics of a 2×2 planar array of equilateral triangular patch microstrip antenna built up on a ferrite substrate $\text{Ni}_{1.062}\text{Co}_{0.02}\text{Fe}_{1.948}\text{O}_4$. Design requirement and substrate characteristics considered for this analysis are listed in Table I.

2. Radiation field expressions

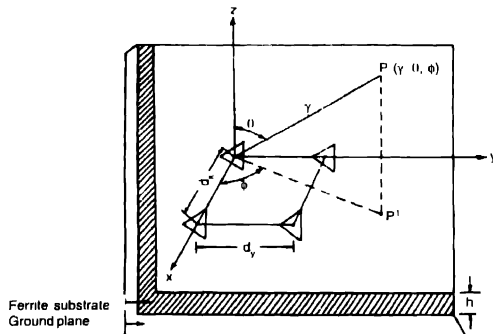
The geometry and coordinate system of planar array antenna is shown in Figure 1. It consists of four identical triangular microstrip patch elements of arm length a , on a ferrite substrate $\text{Ni}_{1.062}\text{Co}_{0.02}\text{Fe}_{1.948}\text{O}_4$ of thickness h and substrate permittivity ϵ_r . The array elements which are positioned along x -axis are separated by a distance d_x and those along y -axis are separated by a distance d_y . Each patch can be excited by a microstrip transmission line connected to the edge or by a coaxial line from the back at the plane $\phi = 0$. We have considered the patch as

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Table 1. Design requirement and substrate characteristics of ferrite substrate $\text{Ni}_{1.062}\text{Cr}_{0.02}\text{Fe}_{1.948}\text{O}_4$

Design frequency (f)	1.0 GHz
Relative permittivity (ϵ_r)	14.78
Dielectric loss tangent ($\tan \delta_r$)	0.0005
Magnetic loss tangent ($\tan \delta_m$)	0.005
Applied d.c. magnetic bias field (H_0)	7.96×10^4 Amp/m
Saturation magnetization ($\mu_0 M_s$)	0.03 Tesla
Patch dimension (a)	0.022 meter
Substrate thickness (h)	0.0016 meter
Gyromagnetic ratio (γ)	1.76×10^{11} rad/sec Tesla
Separation between the array element along x- and y-axis respectively ($d_x = d_y$)	$\lambda/2$
Progressive phase excitation difference along x- and y-axis respectively (β_x, β_y)	$\pi/2$

a cavity which acts as a disc resonator. Among the various modes that may be excited in such disc resonator, we have considered TM_{nm} mode with respect to z-axis. Here, m and n are the mode numbers associated with x- and y- directions respectively

**Figure 1.** Configuration and coordinate system of 2x2 element planar array equilateral triangular patch antenna on ferrite substrate

The E_z component of field inside the cavity for dominant mode is given as

$$E_z = A_{1,0} \left[2 \cos \left(2\pi x / \sqrt{3}a + 2\pi/3 \right) \cos(2\pi y/3a) + \cos(4\pi y/3a) \right] \quad (1)$$

A ferrite slab may support various guided modes and these propagating modes get affected in the presence of applied magnetic bias field. A strong bias field may be obtained easily along the plane of antenna by placing two pole pieces of magnet

in close contact with substrate. When propagation of electromagnetic waves take place along this bias field, infinite ferrite having two orthogonal feeds, two polarized modes namely left hand circularly polarized (LHCP) mode and right hand circularly polarized (RHCP) mode are generated.

The magnetic properties of ferrite substrate affect both modes, therefore the effective permeability for right hand circular polarization and left hand circularly polarization can be given as [6-8]

$$\mu_{effR} = \mu_0 \frac{1 + \omega_m}{\omega_0 - \omega}$$

$$\mu_{effL} = \mu_0 \frac{1 + \omega_m}{\omega_0 + \omega}$$

where ω_0 and ω_m are the precession and forced precession frequencies, respectively and may be defined as

$$\omega_0 = \mu\gamma H_0, \omega_m = \mu\gamma H_s, \text{ and } \omega = 2\pi f$$

The expression of the resonant frequency of concentric equilateral triangular microstrip antenna on ferrite substrate TM_{10} mode is given as

$$f_{r10} = \frac{2}{3a} \frac{\epsilon \sqrt{\mu_0}}{\sqrt{\epsilon_r \mu_{eff}}}$$

Other field components are obtained by solving Maxwell equations. By image theory, the ground plane may be replaced by an image of the top conductor. The magnetic currents exist along the edges of triangular conductor and may be evaluated from

$$\mathbf{M} = 2(\mathbf{E} \times \mathbf{n})$$

where \mathbf{n} is a unit vector normal to the aperture

The total far zone fields of planar array antenna can be expressed by the fields of a single element positioned at origin multiplied by a factor which is referred to as the array factor. This method is widely known as pattern multiplication approach. Thus, using the expression of single element equilateral triangular patch microstrip antenna and neglecting the coupling between the elements, the total field of N element array antenna can be expressed as [9-12]

$E(\text{total}) = E(\text{single element placed at the origin}) \times \text{array factor (AF)}$

As the entire array is taken as uniform, the normalized value of the array factor (AF) is obtained and may be written as

$$AF_n(\theta, \phi) = \frac{1}{M} \times \frac{\sin(M\psi_x/2)}{\sin(\psi_x/2)} \times \frac{1}{N} \times \frac{\sin(N\psi_y/2)}{\sin(\psi_y/2)}$$

For the case of 2×2 element planar array of equilateral triangular patch microstrip antenna on ferrite substrate, the EM p mode fields are given as

EM mode

$$E_{\theta} = \eta_0 \omega \left[-F_x \sin \phi + F_y \cos \phi \right] \times \frac{1}{4} \times \frac{\sin(\psi_x)}{\sin(\psi_x/2)} \times \frac{\sin(\psi_y)}{\sin(\psi_y/2)} \quad (8)$$

$$E_{\phi} = \eta_0 \omega \left[F_x \cos \theta \cos \phi + F_y \cos \theta \sin \phi \right] \times \frac{1}{4} \times \frac{\sin(\psi_x)}{\sin(\psi_x/2)} \times \frac{\sin(\psi_y)}{\sin(\psi_y/2)} \quad (9)$$

$$E_{\theta} = 2hf\beta_p \omega_p^2 / 3a\omega\epsilon_0 (\omega^2 - \omega_p^2) \times \exp(i\beta_p r) / \exp \left[-\beta_p a \sin \theta \cos \phi / \sqrt{3} \right] \times \frac{1}{4} \times \frac{\sin \left\{ \beta_p d_x \sin \theta \cos \phi + \beta_x \right\}}{\sin \left\{ 0.5 (\beta_p d_x \sin \theta \cos \phi + \beta_x) \right\}} \times [E_{\theta} + E_{\phi}] \quad (10)$$

$\psi_x = \beta_x d_x \sin \theta \cos \phi + \beta_x$ and $\psi_y = \beta_y d_y \sin \theta \cos \phi + \beta_y$, M, N = elements placed along the x-axis and y-axis respectively, β_x, β_y = progressive phase excitation along x- and y-directions respectively, E_{θ}, E_{ϕ} = components of total electric field vectors for EM-mode, E_{θ}, E_{ϕ} = electric field vector for P-mode, F_x, F_y = vector electric field for x-component and y-component, respectively,

E_{θ}, E_{ϕ} = P-mode electric field vectors for x-component and y-component respectively, β_p = phase propagation constant for EM-mode given by $2\pi A/\lambda_0$, β_p = phase propagation constant for P-mode given by $\beta_p c/v$, c = velocity of light, v = root mean square thermal velocity of electron, A = plasma frequency parameter given by $(1 - \omega_p^2/\omega^2)^{1/2}$, ω_p = angular plasma frequency, ω = angular source frequency, η_0 = free space impedance equal to 120π ohms

3. Field patterns

The expression for total field pattern $R(\theta, \phi)$ is obtained as

$$R(\theta, \phi) = |E_{\theta}|^2 + |E_{\phi}|^2 \quad (11)$$

Then, the radiation field patterns in the E-plane ($\phi = 0$) and H-plane ($\phi = \pi/2$) are given as

$$R_e(\theta, \phi) = |E_{\theta}|^2 + |E_{\phi}|^2 = \eta_0^2 \omega^2 \left(|F_x|^2 + |F_y|^2 \cos^2 \theta \right), \quad (\text{E-plane}) \quad (12)$$

$$R_h(\theta, \phi) = |E_{\theta}|^2 + |E_{\phi}|^2 = \eta_0^2 \omega^2 \left(|F_x|^2 + |F_y|^2 \cos^2 \theta \right) \quad (13)$$

The values of R_e and R_h are calculated for a case taking $f = 1$ GHz, $a = 2.2$ cm, $\epsilon_r = 14.78$, Applied d.c. magnetic bias field (H_0) = 7.96×10^4 Amp/m, Saturation magnetization ($\mu_0 M_s$) = 0.03 Tesla, $d_x = d_y = \lambda/2$ and the phase difference $\beta_x = \beta_y = \pi/2$. The results are plotted in Figures 2 and 3 for two different planes ($\phi = 0$ and $\phi = \pi/2$) for $A = 1.0$ in free space and in Figures 4 and 5 for two different planes ($\phi = 0$ and $\phi = \pi/2$) $A = 0.5$ in plasma medium for TM₁₀ mode of excitation. Pattern characteristics of planar array are computed in Table 2. Comparison between radiation properties of the planar array antenna on dielectric and ferrite substrate are shown in Table 3.

Table 2 Pattern characteristics of the 2×2 element planar array of triangular microstrip patch antenna on ferrite substrate

Sr No	Pattern characteristics	$\phi = 0$ plane $A = 1.0$		$\phi = 0$ plane $A = 0.5$		$\phi = \pi/2$ plane $A = 1.0$		$\phi = \pi/2$ plane $A = 0.5$	
		RHCP	LHCP	RHCP	LHCP	RHCP	LHCP	RHCP	LHCP
1	Half power beam width (HPBW)	60°	80°	140°	150°	60°	80°	140°	150°
2	Full null beam width (FNBW)	120°	140°	180°	280°	120°	140°	180°	280°
3	Direction of max radiation	90°	90°	180°	180°	90°	90°	180°	180°

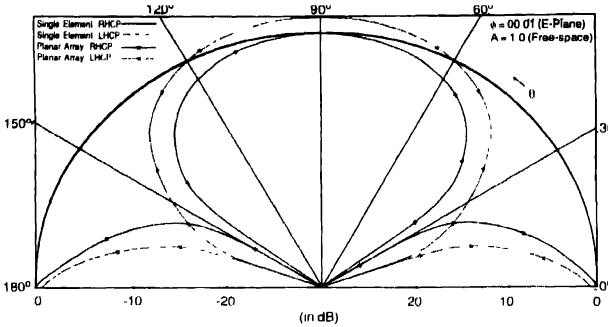


Figure 2. E plane radiation patterns of 2x2 element planar array and single-element equilateral triangular patch microstrip antennas on ferrite substrate for $A = 1.0$ (free space)

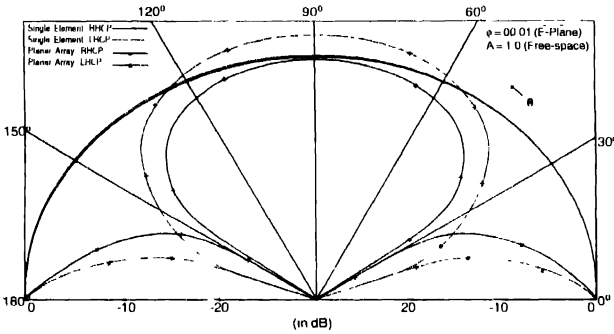


Figure 3. H-plane radiation patterns of 2x2 element planar array and single-element equilateral triangular patch microstrip antennas on ferrite substrate for $A = 1.0$ (free space)

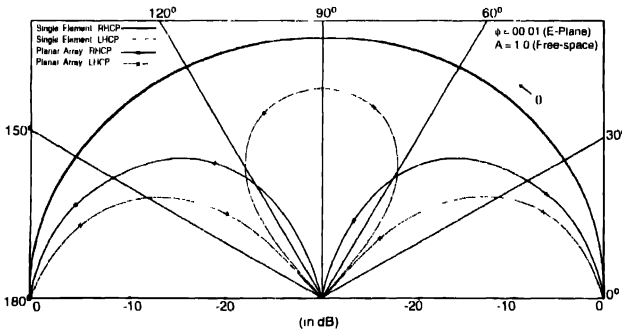


Figure 4. E-plane radiation patterns of 2x2 element planar array and single-element equilateral triangular patch microstrip antennas on ferrite substrate for $A = 0.5$ (plasma)

4. Other antenna parameters

4.1 Radiation conductance

The total power radiated can be calculated by employing Poynting's theorem and P_r as

$$P_r = \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) \left\{ \text{Re} \iint_S (E \times H^*) \cdot d\mathbf{S} \right\}$$

The factor $\frac{1}{2}$ is due to the fact that the power is radiated through the upper half space and S is the total spherical surface area

$$P_r = \left(\frac{1}{4}\right) \left\{ \text{Re} \iint_S (E_\theta H_\phi^* - E_\phi H_\theta^*) \cdot d\mathbf{S} \right\}$$

Thus, the expression for radiated power in EM-modes is obtained using the relationship

$$P_r = \left(\frac{A}{4\eta_0}\right) \int_0^{2\pi} \int_0^\pi |E_\theta|^2 \sin\theta \, d\theta \, d\phi$$

Here, $A = \left(1 - \omega_p^2/\omega^2\right)^{1/2}$ and ω_p is the plasma to source frequency ratio

The radiation conductance of an antenna EM-mode can be defined as

$$G_r = 2 P_r / V_0^2$$

4.2 Directivity

The directivity of an antenna is defined as ratio of the maximum radiation intensity (per unit solid angle) to the average radiation intensity. It can be expressed as

$$D_r = 4\pi \left\{ \max \left(|E_\theta|^2 + |E_\phi|^2 \right) \right\} \left\{ \int_0^{2\pi} \int_0^\pi |E_\theta|^2 + |E_\phi|^2 \sin\theta \, d\theta \, d\phi \right\}^{-1}$$

4.3 Quality factor

A parameter specifying frequency selectivity of a resonant circuit is the quality factor Q , which can be defined as the ratio between energy stored in the system and the energy lost

The total Q of a microstrip radiating element comprises contributions due to the radiation Q_r , conductor loss Q_c , and dielectric loss Q_d quality factors, so

Table 3. Comparison between radiation properties of the 2×2 element planar array of triangular microstrip patch antenna on pure dielectric substrate and ferrite substrate in TM_{10} mode

	Patch dimensions (meters)	Resonant frequency (GHz)	Radiation conduction (mW)	Quality factor	Directive gain (dB)
Pure-dielectric substrate $\epsilon_r = 2.32$, $h = 0.0016$	0.022	5.97	0.003	7.756	6.518
Ferrite substrate, LHCP $\epsilon_r = 14.78$, $h = 0.0016$	0.022	1.2	1.116×10^{-4}	252.15	7.5
Ferrite substrate, RHCP $\epsilon_r = 14.78$, $h = 0.0016$	0.022	0.9	7.356×10^{-5}	259.09	6.625

$$1/Q = 1/Q_r + 1/Q_c + 1/Q_d, \quad (17)$$

$$Q_c = \omega W_c / P_c,$$

$$Q_r = \omega W_r / P_r = (\pi f \mu \sigma)^{1/2} h,$$

$$Q_d = \omega W_d / P_d = 1/\tan \delta$$

Let: W is the energy stored in the antenna element, P_c and P_r are the power loss factors due to the conductors and dielectric, respectively. σ is the conductivity of the conductors.

Energy stored in the triangular radiating element is given

$$W = (1/2) \int_V \epsilon |\mathbf{E}|^2 dV \quad (18)$$

Conclusion

The radiation characteristics of 2×2 planar array of equilateral triangular patch microstrip antenna on ferrite substrate in plasma

medium have been studied by considering the presence of a bias magnetic field in the direction of propagation of electromagnetic waves. The results of the array geometry are compared with those of single-element of equilateral triangular patch microstrip antenna. Figures 2 and 3 illustrate the radiation patterns of 2×2 element planar array of equilateral triangular microstrip antenna for two planes ($\phi = 0$ and $\phi = \pi/2$) in free space. It is observed that the beam splits up into major and minor lobes and corresponding position of maximum radiation is shifted. The radiation patterns of a single element antenna for both RHCP and LHCP waves contain only one major lobe of considerably wide beam width, while the array geometry for both RHCP and LHCP waves produces a directive beam with a narrow beam width. The radiation field patterns of this array antenna indicate that beam width with LHCP waves is larger than that with RHCP waves. Figures 4 and 5 illustrate the radiation patterns of 2×2 element planar array of equilateral triangular microstrip antenna for two planes ($\phi = 0$ and $\phi = \pi/2$) in plasma medium ($A=0.5$). From these figures, it is observed

that the shape of the field patterns has been changed and it redistributes the field intensity. The computed values of pattern characteristics of the planar array are given in Table 2. It is clear from this table that half power beam width of the patterns in free space is relatively small in comparison to plasma medium. Thus it reveals that the patterns are more directive in free-space than in plasma medium. From Tables 4 and 5, it is clear that the effect of plasma on radiation conduction of single element as well as planar array of equilateral triangular microstrip antenna in RHCP mode is significantly weaker than the LHCP mode. The directivity of 2×2 element planar array of equilateral triangular microstrip antenna with LHCP waves is little higher than

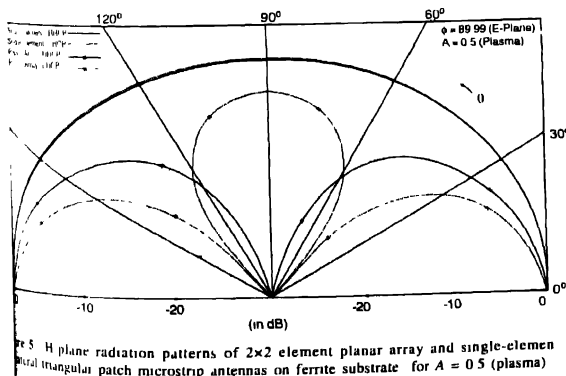


Table 4. Antenna parameters for single element and 2×2 planar array equilateral triangular patch microstrip antenna on ferrite substrate with RHCP waves

Sl. No	Plasma parameter (A)	G_s for single element (mho)	G_s for 2×2 planar array (mho)	D_s for single element (dB)	D_s for 2×2 planar array	Q_s for single element	Q_s for 2×2 planar array
1	1.0	1.588×10^{-5}	7.356×10^{-5}	4.751	6.625	444.418	259.806
2	0.9	1.428×10^{-5}	6.819×10^{-5}	4.755	6.414	453.355	270.265
3	0.8	1.268×10^{-5}	6.152×10^{-5}	4.758	6.244	462.635	284.459
4	0.7	1.109×10^{-5}	5.379×10^{-5}	4.761	6.108	472.281	302.933
5	0.6	9.468×10^{-6}	4.537×10^{-5}	4.764	5.99	482.319	325.981
6	0.5	7.911×10^{-6}	3.673×10^{-5}	4.766	5.885	492.774	353.601
7	0.4	4.743×10^{-6}	2.829×10^{-5}	4.768	5.74	503.678	385.479
8	0.3	4.743×10^{-6}	2.073×10^{-5}	4.769	5.523	515.061	421.119
9	0.2	3.161×10^{-6}	1.309×10^{-5}	4.77	5.248	526.961	460.222
10	0.1	1.58×10^{-6}	6.379×10^{-6}	4.771	4.931	539.417	503.304
11	0.0	0.0	0.0	4.771	4.772	-	-

Table 5. Antenna parameters for single element and 2×2 planar array equilateral triangular patch microstrip antenna on ferrite substrate with LHCP waves

Sl. No	Plasma parameter (A)	G_s for single element (mho)	G_s for 2×2 planar array (mho)	D_s for single element (dB)	D_s for 2×2 planar array	Q_s for single element	Q_s for 2×2 planar array
1	1.0	2.811×10^{-5}	1.116×10^{-4}	4.736	7.5	449.784	252.13
2	0.9	2.526×10^{-5}	1.071×10^{-4}	4.742	7.151	462.144	258.296
3	0.8	2.242×10^{-5}	1.013×10^{-4}	4.748	6.767	475.146	266.627
4	0.7	1.959×10^{-5}	9.287×10^{-5}	4.754	6.475	488.847	279.728
5	0.6	1.678×10^{-5}	8.142×10^{-5}	4.758	6.24	503.311	299.766
6	0.5	1.397×10^{-5}	6.751×10^{-5}	4.762	6.063	518.612	328.353
7	0.4	1.117×10^{-5}	5.237×10^{-5}	4.765	5.921	534.831	366.556
8	0.3	8.37×10^{-6}	3.741×10^{-5}	4.768	5.737	552.062	413.675
9	0.2	5.578×10^{-6}	2.364×10^{-5}	4.77	5.435	570.413	469.492
10	0.1	2.788×10^{-6}	1.133×10^{-5}	4.771	5.024	590.006	533.852
11	0.0	0.0	0.0	4.771	4.77	-	-

that with RHCP waves for all the plasma frequency and free space. The total quality factor of planar array antenna with LHCP waves is lower than that with RHCP waves

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